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Waheed Siddiqee
Stanford Research Institute
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# Analysis of Some Packet Radio Networks with Stations

## I Introduction

In this note we consider a few alternative packet radio network schemes with stations. The note can be regarded as a counter part to the note by Peter J. Wong[1] in which a few stationless schemes were studied. The analysis in both notes is based on several simplifying assumptions since the intent was to quickly obtain some broad views of the network with stations and without stations. The results of analyses in both notes pertain to a network of a few repeaters each having a set of terminals associated with it. The terminals associated with each repeater are assumed to transmit a total of $r_0$ and receive a total of $r_0$ packets of length T per unit time. The analysis is concerned essentially with calculating the maximum value of "$r_0T$" under various schemes and protocols of repeaters and stations. Also calculated are the relative amounts of time spent by various repeaters in hearing and transmitting packets from own terminals, packets from other repeaters and in retransmitting the packets (because of unsuccessful attempts). The results are summarized in Section IV. Detailed analysis is presented in the Appendices.

## II Assumptions

We make the following assumptions with reference to all schemes with stations:

1. The station is in the center. 6 repeaters in a hexagonal pattern form the first ring and 12 repeaters the second ring as shown in Figure 1. The results are extendable to the case

1. P. J. Wong, "Channel Utilization of a Scout/Follower Broadcast System," Packet Radio Temporary Note 31, February 26, 1973.

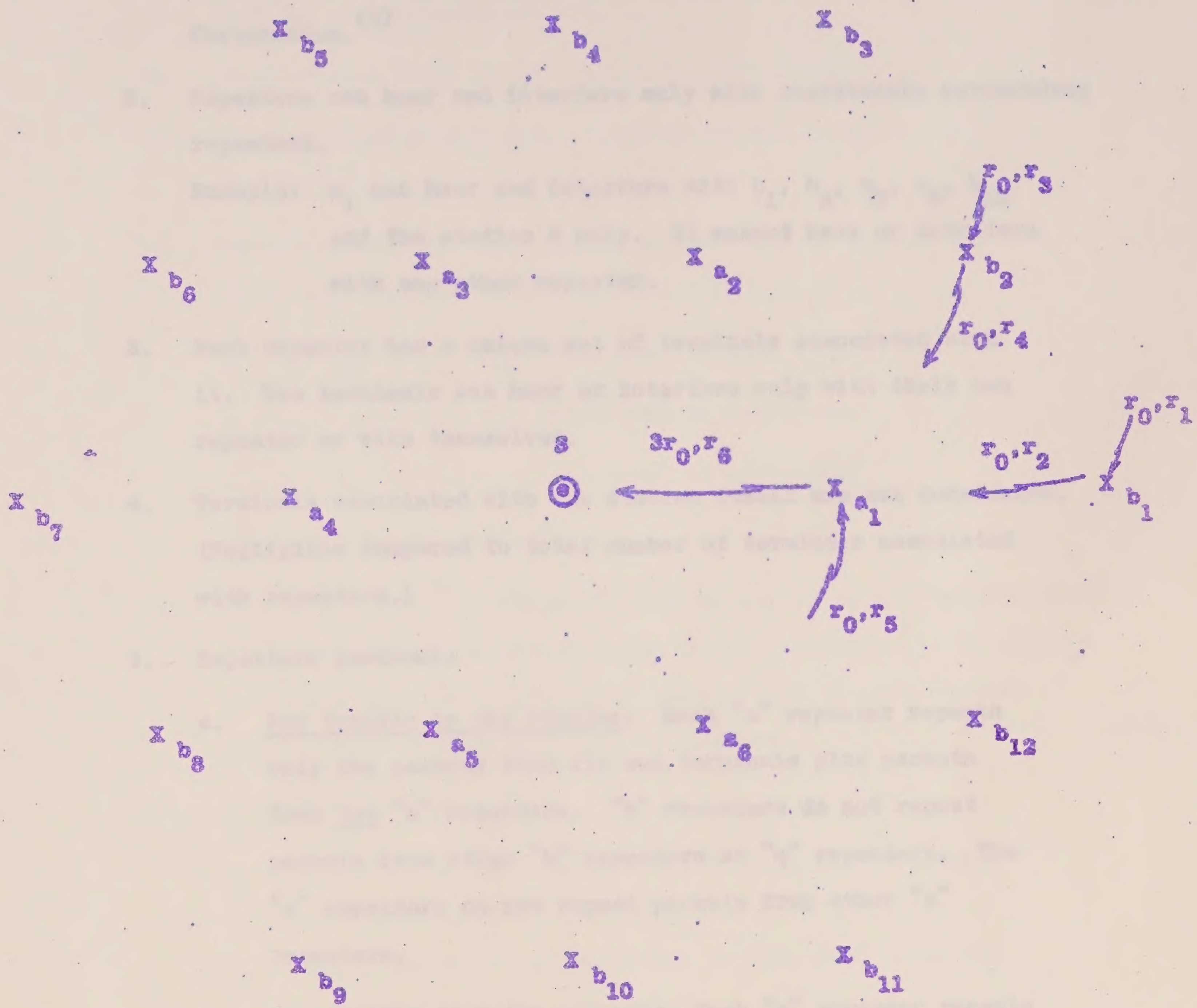


Fig. 1. NAC Type Scheme with Two Rings of Repeaters (Traffic to the Station)

$a_1$, $a_2$, --- are level 1 repeaters (one unit distance from station)

$b_1$, $b_2$, --- are level 2 repeaters (two unit distance from station)

NOTE: $r_1$, $r_2$, $r_3$, etc., are rates of retransmitted packets from various sources

of 3 rings of repeaters (36 repeaters). All traffic generated by various terminal is routed to the station (via repeaters) as shown in Figure 1. Then it is routed to the respective destination terminals (via repeaters) as shown in Figure 2 on page 8. This corresponds to the general philisophy proposed by Network Analysis Corporation.[2]

2. Repeaters can hear and interfere only with immediately surrounding repeaters.

   Example: $a_1$ can hear and interfere with $b_1$, $b_2$, $a_2$, $a_6$, $b_{12}$ and the station S only. It cannot hear or interfere with any other repeater.

3. Each repeater has a unique set of terminals associated with it. The terminals can hear or interfere only with their own repeater or with themselves.

4. Terminals associated with the station itself are not considered. (Negligible compared to total number of terminals associated with repeaters.)

5. Repeaters protocol:

   a. <u>For traffic to the station:</u> Each "a" repeater repeats only the packets from its own terminals plus packets from <u>two</u> "b" repeaters. "b" repeaters do not repeat packets from other "b" repeaters or "a" repeaters. The "a" repeaters do not repeat packets from other "a" repeaters.

   b. <u>For traffic from the station:</u> Each "a" repeater repeats only the packets meant for its own terminals plus packets for two "b" repeaters. The "a" repeaters do not repeat packets from "b" repeaters or other "a" repeaters; and "b" repeaters do not repeat packets from other "b" repeaters.

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2. Network Analysis Corporation, "Packet Radio--System Considerations," Packet Radio Temporary Note #5, January 1, 1973.

6. The terminals in a repeater area generate a total of "$r_0$" packets and receive a total of "$r_0$" packets per unit time, i.e., the total useful network packets = $n\ r_0$ where n is the number of repeaters.

7. Link by link acknowledgement.

8. Interarrival times of all packet is exponential.

9. Transmissions from a given repeater cannot interfere with themselves.

10. Repeaters that are symmetrical in relation to the station and their surroundings behave identically, e.g.,

$b_1$, $b_3$, $b_5$, ------ $b_{11}$ identical to each other

$b_2$, $b_4$, $b_6$, ------ $b_{12}$ identical to each other

$a_1$, $a_2$, $a_3$, ------ $a_6$ identical to each other

11. Slotted channel utilization.

12. As done by Wong[(1)], the acknwoledgement packets are infinitesimally small, implying non-interference between acknowledgement packets and other packets. Also it is assumed that acknowledgement packets are received with probability one.

13. Referring to Figure 1,

$r_1$ = Total number of retransmitted packets from terminals associated with $b_1$ type repeaters

$r_3$ = Total number of retransmitted packets from terminals associated with $b_2$ type repeaters

$r_2$ = Total number of retransmitted packets from $b_1$ type repeaters

$r_4$ = Total number of retransmitted packets from $b_2$ type repeaters

$r_5$ = Total number of retransmitted packets from terminals associated with each "a" type repeater

$r_6$ = Total number of retransmitted packets from each "a" type repeater.

Referring to Figure 2, the meaning of $r_7$, $r_8$, $r_9$, ----, etc., is similarly to be interpreted.

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Assumption 1 is based on the observation that (assuming a circular range of coverage from each repeater) a hexagonal pattern requires the minimum number of repeaters to cover a given area. Also in a hexagonal pattern all the repeaters immediately surrounding any repeater are all equidistant from the center repeater, thus providing a symmetrical pattern.

Assumptions 2, 3, 4, 7, 8 and 12 have been made basically to simplify the analysis. It is planned to consider more complex cases later.

Assumption 5, that each "a" repeater handles its own plus the traffic from two b repeaters, is made so that each "a" repeater is loaded "equally." Another part of the assumption is a protocol assumed to minimize unnecessary retransmissions of the same message from more than one repeater.

Assumption 6 implies a uniform exchange of traffic between various sets of terminals. Again this assumption is made to simplify the analysis plus the fact that the exact traffic flow has not yet been defined. Assumptions 9 and 10 are self-explanatory. The 11th assumption is made on the observation that in an Aloha type of system a slotted system results in doubling the channel utilization compared to a random Aloha. It is thus reasonable to assume during the preliminary analysis that a slotted channel utilization will most probably be superior to a non-slotted in the cases under consideration. However, later a detailed analysis is planned to justify this assumption more rationally.

## III General Description of Various Cases

We shall consider the following four cases:

Case 1: Available channel split into two non-interfering parts. One part for traffic going to the station, the other for traffic going away from the station. No cooperation among repeaters and station, i.e., neighboring repeaters compete for available slots; example: $b_1$, $b_2$, $b_{12}$, and $a_1$ can generate traffic simultaneously.

Case 2: Split channel as above but neighboring repeaters listen before transmitting, i.e., cooperative neighboring repeaters; example: When $b_1$ is transmitting, $b_2$, $b_{12}$, and $a_1$ do not generate any traffic. However, $a_2$ or $a_6$ may generate since they cannot hear $b_1$.

Case 3: Split channel as above. The 6 repeaters surrounding the station operate in total cooperation, i.e., when any "a" repeater is transmitting, no other "a" repeater generates any traffic. However, "b" repeaters can generate conflicting traffic.

Case 4: Single channel for both the traffic going to and away from the station. No cooperation among repeaters.

The following general description about various sets of terminals, repeaters and the station is applicable to all four cases. We consider one repeater of each category (refer to assumption 10) and briefly describe their role in traffic handling. Other repeaters of the same category are assumed to operate in a similar manner.

Considering first the traffic to the station and referring to Figure 1, we have the following pattern:

- $b_1$ type repeater

  The repeater $b_1$ has two level 2 repeaters ($b_2$ and $b_{12}$) and one level 1 repeater ($a_1$) in its vicinity. Considering the traffic to the station, $b_1$ is being offered ($r_0$+$r_1$) packets from its terminals. It is transmitting a total of ($r_0$+$r_2$) packets intended for $a_1$. $r_1$ is the number of retransmissions from terminals associated with $b_1$ and $r_2$ is the number of retransmissions from the repeater $b_1$ to $a_1$.

- $b_2$ type repeater

  The repeater $b_2$ is surrounded by two level 2 repeaters ($b_1$ and $b_3$) and two level 1 repeaters ($a_1$ and $a_2$). It is being offered

$(r_0+r_3)$ packets from its terminals and is transmitting a total of $(r_0+r_4)$ packets intended for $a_1$ where $r_3$ and $r_4$ are respectively retransmissions from terminals associated with $b_2$ and from repeater $b_2$ to $a_1$.

- $a_1$ type repeater

  The repeater $a_1$ is surrounded by three level 2 repeaters ($b_1$, $b_2$, and $b_{12}$) and two level 1 repeaters ($a_2$ and $a_6$). According to our assumption, it is responsible for transmitting traffic from its own terminals $(r_0+r_5)$ and from (say) $b_1$ and $b_2$, i.e., it is transmitting $3r_0$ real packets and $r_6$ retransmissions in its attempt to reach the station.

Considering now the traffic from the station and referring to Figure 2, we have the following pattern:

- The station

  The station is transmitting $6(3r_0+r_7)$ packets, assuming $3r_0$ for each "a" repeater, $r_7$ being the number of retransmissions from the station associated with each "a" repeater.

- $a_1$ type repeater

  The $a_1$ repeater is transmitting $(r_0+r_{10})$ packets to its terminals, $(r_0+r_8)$ packets to $b_1$ repeater and $(r_0+r_9)$ packets to $b_2$ repeater. $r_8$, $r_9$, and $r_{10}$ are the retransmissions.

- $b_1$ and $b_2$ type repeater

  The $b_1$ and $b_2$ repeaters each are transmitting $(r_0+r_{11})$ and $(r_0+r_{12})$ packets to their respective terminals, $r_{11}$ and $r_{12}$ being the retransmissions.

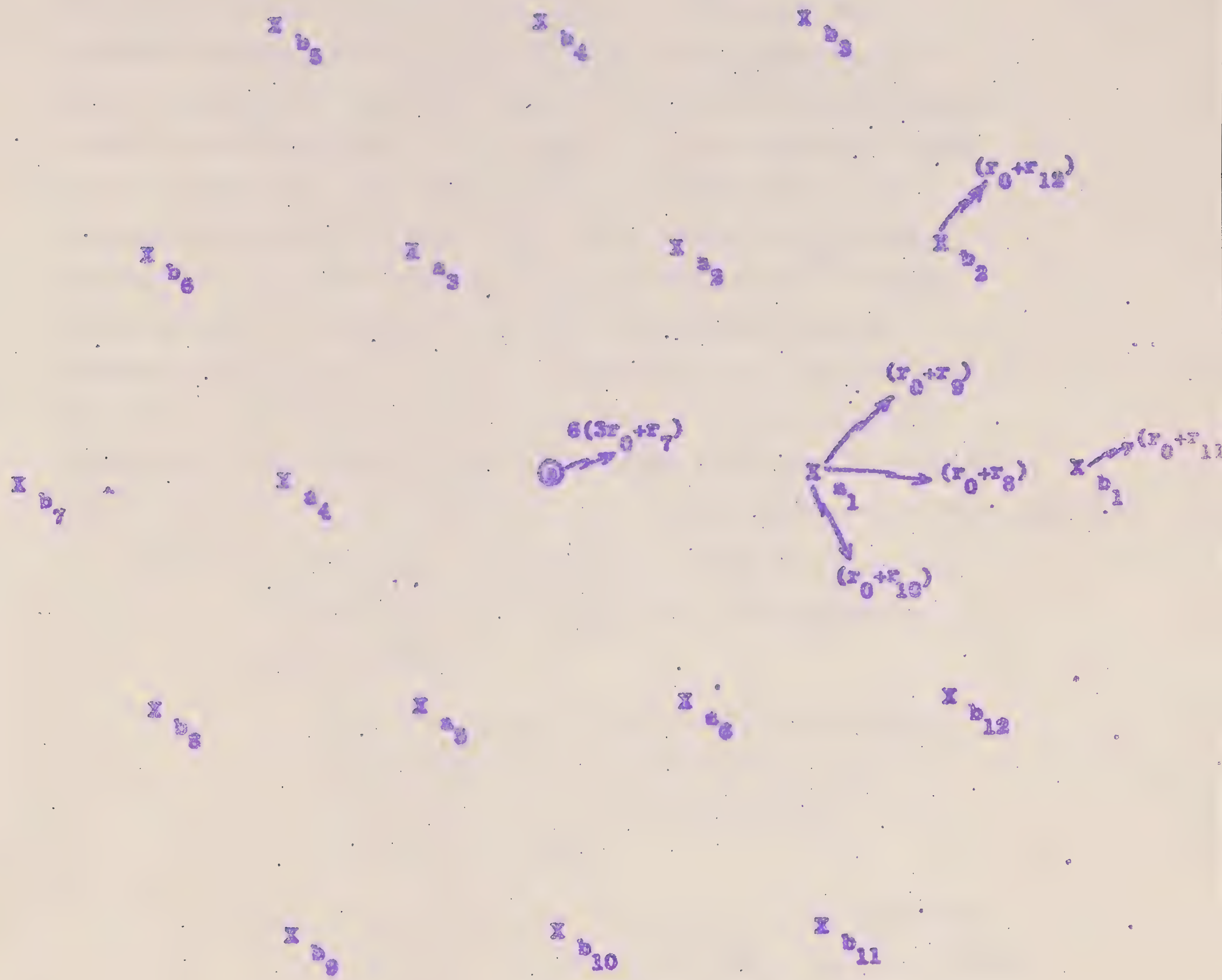


Fig. 2. Traffic from the Station

## IV Summary of Results

In this section we summarize the results of the analysis of the four cases mentioned in Section III. The detailed development of equations and their solutions are included in the Appendices. Two types of results are summarized. First, the product of packet length T and the maximum possible useful traffic "$n\, r_0$" for the net is indicated for each case. The useful traffic per repeater area is then obtained by simply dividing the total useful traffic by the number of repeaters. For convenience of comparison with Wong's[1] results (refer to Page 2 of his note), the traffic per repeater area is calculated for 36 repeater networks. The second set of results indicate the pattern of the behavior of the repeaters of the inner ring ("a" repeaters). These are also roughly comparable to the numbers indicated under item (iii), page 2 of Wong's[1] note. Station performance is also indicated.

Case 1. Split channel (2/3 channel for traffic to the station, 1/3 for traffic from the station). No cooperation among repeaters.

(i) The maximum useful traffic which can be offered by the net, i.e., the maximum value "$n\, r_0 T$" is 0.293 where n is the number of repeaters. For $n = 36$, $r_0 T = .0081$.

(ii) • Of the total traffic offered to and transmitted by an "a" repeater, about 20% accounts for packets from own terminals, about 28% for transit traffic (from its two "b" repeaters) and 52% in listening to neighboring repeaters (i.e., listening to one "b" and two "a" repeaters whose traffic is not its responsibility.

• The station spends about 58% of its time in handling useful traffic, the rest, 42%, is wasted in retransmissions and listening to packets which are not its responsibility.

Case 2. Split channel (1/2 channel for traffic to the station, 1/2 channel for traffic from the station). Repeaters listen before transmitting.

The listening before transmitting principle can be elaborated by an example. Referring to Fig. 1 and considering the traffic $(r_0+r_2)$ from $b_1$, the listen before transmit principle implies that repeaters $b_2$, $b_{12}$ and $a_1$ will not interfere with $(r_0+r_2)$ since these repeaters are within the listening range of $b_1$. Only $a_2$ and $a_6$ and the traffic $(r_0+r_5)$ will interfere with $(r_0+r_2)$.

(i) Maximum value of n $r_0T$ is 0.36. For n = 36, $r_0T$ = .01.

(ii) • Of the total traffic at an "a" repeater, about 18% accounts for packets from own terminals, 28% for transit traffic (from two b repeaters) and 50% for listening to neighboring repeaters.

• Station spends about 72% of its time in handling useful traffic. The rest, 28%, is wasted in retransmissions and listening to packets not its responsibility.

Case 3. Split channel (2/5 channel for traffic to the station, 3/5 for traffic from the station). Total cooperation among inner ring of repeaters. (Implies that when one "a" repeater is transmitting, no other "a" repeater will interfere.)

(i) The maximum value of n $r_0T$ = 0.4; for n = 36, $r_0T$ = .011.

(ii) • Of the total traffic at an "a" repeater, 24% accounts for packets from own terminals, about 32% for transit traffic and 44% for listening to neighboring repeaters.

• Station spends 80% of its time in handling useful traffic.

Case 4. Single channel. No cooperation among repeaters.

Single channel implies that the traffic "to" and "from" the station uses the same channel.

(i) Maximum value of $n\ r_0 T = 0.182$. For $n = 36$, $r_0 T =$ .005.

(ii) • Of the total traffic at an "a" repeater, 41% accounts for packets of own terminals plus transit traffic. A breakup of own and transit traffic has not been calculated for this case. 59% accounts for listening to neighbors.

• Station spends about 38% time in handling useful traffic.

The results of the above four cases are summarized in a concise form in the following table:

Table 1

CONCISE SUMMARY OF RESULTS

| Case | Max. "$n r_0 T$" | "a" Repeater Performance (% of Total Traffic Handled) | | | Station Performance (Time Spent in Handling Useful Traffic) |
|---|---|---|---|---|---|
| | | Own Terminals | Transit Traffic | Interfering Traffic | |
| 1. Split channel. No cooperation. | .293 | 20 | 28 | 52 | 58% |
| 2. Split channel. Cooperative neighboring repeaters. | .36 | 16 | 28 | 56 | 72% |
| 3. Split channel. Total cooperation among "a" repeaters. | .4 | 24 | 32 | 44 | 80% |
| 4. Single channel. No cooperation. | .192 | 41 | | 59 | 38% |

## Appendix A

## Equations and Their Solutions for Case 1

We now consider Case 1 in some detail. For this case we first develop the equations in Part A, summarize the equations in Part B and present the solution in Part C. In Part D, we briefly explain the results.

### Case 1. Split Channel. No Cooperation Among Repeaters

### A. Development of Equations

First let us consider the traffic going to the station (refer to Fig. 1). Since split channel is assumed, the traffic to the station will not interfere with the traffic from the station.

- Consider the traffic $(r_0+r_1)$

  In addition to $(r_0+r_1)$, other transmissions which can conflict with $(r_0+r_1)$ are:

  $(r_0+r_2)$ = the transmissions from $b_1$

  $(r_0+r_4)$ = the transmissions from $b_2$

  $(r_0+r_4)$ = the transmissions from $b_{12}$

  $(3r_0+r_6)$ = the transmissions from $a_1$

  Thus the total busy time at $b_1$, with reference to $(r_0+r_1)$ is proportional to:

  $$R_1 = (r_0+r_1) + (r_0+r_2) + 2(r_0+r_4) + (3r_0+r_6) \qquad (1)$$

  and following Abramson's[2] approach:

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2. Norman Abramson, "The Aloha System - Another Alternative for Computer Communications," AFIPS Conference Proceedings, Vol. 37, pp. 281-285, Nov. 1970.

$$r_1 = (r_0+r_1)\,[1 - e^{-R_1T}]$$

$$\therefore\ r_0 = (r_0+r_1)\,e^{-R_1T} \qquad (2)$$

- Consider $(r_0+r_3)$

The total busy time, following a similar reasoning as above, is proportional to:

$$R_3 = (r_0+r_3) + (r_0+r_4) + 2(r_0+r_5) + 3(3r_0+r_6) \qquad (3)$$

$$\therefore\ r_3 = (r_0+r_3)\,[1 - e^{-R_3T}]$$

$$\therefore\ r_0 = (r_0+r_3)e^{-R_3T} \qquad (4)$$

- Consider $(r_0+r_2)$ attempting to reach $a_1$

The busy time with reference to $(r_0+r_2)$ is proportional to:

$$R_2 = (r_0+r_5) + 2(r_0+r_4) + 3(3r_0+r_6) \qquad (5)$$

$$\therefore\ r_2 = (r_0+r_2)\,[1 - e^{-R_2T}]$$

$$\therefore\ r_0 = (r_0+r_2)\,e^{-R_2T} \qquad (6)$$

- Consider $(r_0+r_4)$ attempting to reach $a_1$

$$R_4 = (r_0+r_4) + (r_0+r_5) + (r_0+r_2) + 3(3r_0+r_6) \qquad (7)$$

$$\therefore\ r_0 = (r_0+r_4)\,e^{-R_4T} \qquad (8)$$

- Consider $(r_0+r_5)$

$$R_5 = (r_0+r_5) + 2(r_0+r_4) + (r_0+r_2) + 3(3r_0+r_6) \qquad (9)$$

$$\therefore\ r_0 = (r_0+r_5)\,e^{-R_5T} \qquad (10)$$

• Consider $(3r_0+r_6)$

$$R_6 = 5\ (3r_0+r_6) \tag{11}$$

$$\therefore\ r_0 = \frac{(3r_0+r_6)}{3}\ e^{-R_6T} \tag{12}$$

Equations 2, 4, 6, 8, 10 and 12 provide the relationship between $r_0$, $r_1$, $r_2$, ----- $r_6$ for the traffic going to the station.

Considering now the traffic from the station and referring to Fig. 2.

• Consider $(3r_0+r_7)$ trying to reach $a_1$

The conflicting traffic at $a_1$ is

| | |
|---|---|
| $(r_0+r_8)$ + $(r_0+r_9)$ + $(r_0+10)$ | from $a_1$ |
| " " " | from $a_3$ |
| " " " | from $a_6$ |
| $(r_0+r_{11})$ | from $b_1$ |
| $(r_0+r_{12})$ | from $b_2$ |
| $(r_0+r_{12})$ | from $b_{12}$ |

Thus:

$$R_7 = 3\ \{(r_0+r_8) + (r_0+r_9) + (r_0+r_{10})\} + 3(r_0+r_{12}) + (r_0+r_{11}) \tag{13}$$

and $r_7 = (3r_0+r_7)\ (1 - e^{-R_7T})$

$$\therefore\ r_0 = \frac{(3r_0+r_7)}{3}\ e^{-R_7T} \tag{14}$$

• Consider now $(r_0+r_8)$ trying to reach $b_1$

The conflicting traffic at $b_1$ is:

$(r_0+r_{11})$ from $b_1$

$(r_0+r_{12})$ from $b_3$

$(r_0+r_{12})$ from $b_{12}$

Considering now the fact that when repeaters $b_1$, $b_2$, transmit the messages to the terminals, there is no conflicting traffic at the terminals; it can be stated that there will be no retransmissions required for the traffic from repeaters to the terminals. Thus for the case under consideration

$$r_{10} = r_{11} = r_{12} = 0$$

$$\text{and } R_8 = 3r_0 \tag{15}$$

$$\therefore\ r_8 = (r_0+r_8)\ (1 - e^{-3r_0T})$$

$$\therefore\ r_0 = (r_0+r_8)\ e^{-3r_0T} \tag{16}$$

• Consider now $(r_0+r_9)$ trying to reach $b_2$

The conflicting traffic at $b_2$ is:

$r_0$ from $b_3$

$r_0$ from $b_1$

$r_0$ from $b_3$

$(r_0+r_8) + (r_0+r_9) + (r_0+r_{10})$ from $a_2$

$$\therefore \quad R_9 = 3r_0 + (r_0+r_8) + (r_0+r_9) + (r_0+r_{10}) \qquad (17)$$

$$\therefore \quad r_0 = (r_0+r_9)\, e^{-R_9 T} \qquad (18)$$

Equations 14, 16 and 18 provide the relationship between $r_0$, $r_7$, $r_8$ and $r_9$ for the traffic going away from the station.

B. Summary of Equations

- For traffic going to the station (Fig. 1).

$$r_0 = (r_0+r_1)\, e^{-[(r_0+r_1) + (r_0+r_2) + (r_0+r_4) + (3r_0+r_6)]T}$$

$$r_0 = (r_0+r_3)\, e^{-[(r_0+r_3) + (r_0+r_4) + 2(r_0+r_2) + 2(3r_0+r_6)]T}$$

$$r_0 = (r_0+r_2)\, e^{-[(r_0+r_5) + 2(r_0+r_4) + 3(3r_0+r_6)]T}$$

$$r_0 = (r_0+r_4)\, e^{-[(r_0+r_4) + (r_0+r_5) + (r_0+r_2) + 3(3r_0+r_6)]T}$$

$$r_0 = (r_0+r_5)\, e^{-[(r_0+r_5) + 2(r_0+r_4) + (r_0+r_2) + 3(3r_0+r_6)]T}$$

$$r_0 = \frac{(3r_0+r_6)}{3}\, e^{-5(3r_0+r_6)T}$$

(NOTE: $r_{10} = r_{11} = r_{12} = 0$)

- For traffic coming from the station (Fig. 2)

$$r_0 = \frac{(3r_0+r_7)}{3}\, e^{-3[(r_0+r_8) + (r_0+r_9) + 2r_0]T}$$

$$r_0 = (r_0+r_8)\, e^{-3r_9 T}$$

$$r_0 = (r_0+r_9)\, e^{-[(r_0+r_8) + (r_0+r_9) + 4r_0]T}$$

## C. Solution of Equations

We first solve the equations related to the traffic going to the station.

Define $r_0T = x_0$, $(r_1+r_0)T = x_1$, $(r_0+r_2)T = x_2$, $(r_0+r_3)T = x_3$

$(r_0+r_4)T = x_4$, $(r_0+r_5)T = x_5$, $(3r_0+r_6)T = x_6$

The equations can then be written as:

$$x_0 = x_1 e^{-(x_1+x_2+2x_4+x_6)} \tag{C-1}$$

$$= x_3 e^{-(x_3+x_4+2x_2+2x_6)} \tag{C-2}$$

$$= x_2 e^{-(x_5+2x_4+3x_6)} \tag{C-3}$$

$$= x_4 e^{-(x_4+x_5+x_2+3x_6)} \tag{C-4}$$

$$= x_5 e^{-(x_5+2x_4+x_2+3x_6)} \tag{C-5}$$

$$= \frac{x_6}{3} e^{-5x_6} \tag{C-6}$$

At this stage, we can either use Newton's Method to solve the above equations (assuming different values of $x_0$), or solve the equations using suitable trial values taking advantage of the special structure of the equations. We follow the latter method since it is easily applicable here.

First we note that equation (C-6) provides us with a straightforward relationship between $x_0$ and $x_6$. A set of some typical values is:

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_6$ | 0 | .05 | .1 | .2 | .3 | .5 |
| $x_0$ | 0 | .013 | .0202 | .0244 | .0223 | .0135 |

Note that $x_0$ is maximum when $x_6 = 0.3$.

We shall now manipulate equations (C-3), (C-4) and (C-5) in order to eliminate $x_2$ and $x_5$ and obtain an equation in $x_4$, $x_0$ and $x_6$ as follows:

Equation (C-3) can be written as

$$x_0 \, e^{x_5} = x_2 \, e^{-(2x_4+3x_6)} \tag{C-7}$$

Equation (C-4) can be written as

$$x_0 \, e^{x_5} = x_4 \, e^{-(x_4+x_2+3x_6)} \tag{C-8}$$

Equating the right sides of (C-7) and (C-8), we have

$$x_2 \, e^{-(2x_4+3x_6)} = x_4 \, e^{-(x_4+x_2+3x_6)}$$

$$\therefore \; x_2 \, e^{x_2} = x_4 \, e^{x_4}$$

$$\therefore \; \boxed{x_2 = x_4}$$

Now, equation (C-5) can be written as:

$$x_0 \, e^{x_5} = x_5 \, e^{-(2x_4+x_2+3x_6)} \tag{C-9}$$

In (C-9), substitute the right side of (C-8) for $x_0 \, e^{x_5}$ to get:

$$x_4 \, e^{-(x_4+x_2+3x_6)} = x_5 \, e^{-(2x_4+x_2+3x_6)}$$

$$\therefore \; x_4 = x_5 \, e^{-x_4}$$

$$\therefore \; x_4 \, e^{x_4} = x_5 \tag{C-10}$$

From (C-7) we can obtain, by taking log of both sides,

$$x_5 = \log \frac{x_2}{x_0} - 2x_4 - 3x_6 \qquad \text{(C-11)}$$

Thus (C-10) can be written as

$$x_4 e^{x_4} = \log \frac{x_2}{x_0} - 2x_4 - 3x_6$$

$$\boxed{x_4 e^{x_4} = \log \frac{x_4}{x_0} - 2x_4 - 3x_6} \qquad \text{(C-12)}$$

Equation (C-12) has been obtained by eliminating $x_2$ and $x_5$ from equations (C-3), (C-4) and (C-5).

For various sets of values of $x_0$ and $x_6$ (as indicated already), $x_4$ can be calculated directly. Since $x_2$ is equal to $x_4$, we have the solution for $x_2$ as well. Having found $x_2$ and $x_4$ for various sets of $x_0$ and $x_6$, it is a straightforward (though laborious) process to calculate $x_5$ and then $x_3$ and $x_1$ using the equations (C-3), (C-2) and (C-1). For the sake of brevity we only list below a few of the calculated set of values for $x_1$, $x_2$, $x_3$, ..., without going through the details of the calculations. The last row corresponds to the maximum values.

Table 1. Set of Values for Various x's

| | $x_0$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .013 | | .016 | | .018 | .022 | .05 |
| | .0202 | | .03 | | .03 | .05 | .1 |
| Maximum Values | .0244 | .035 | .062 | .045 | .052 | .09 | .2 |

The above set of values can now be translated in terms of $r_0T$, $r_1T$, .... $r_6T$ as indicated in Table 2. The relationship between $r_0$, $r_2$, $r_4$, $r_5$ and $r_6$ is shown also graphically in Fig. A-1.

Table 2. Set of Values for Various $(rT)^s$

| | $r_0T$ | $r_1T$ | $r_2T$ | $r_3T$ | $r_4T$ | $r_5T$ | $r_6T$ |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .013 | | .003 | | .003 | .009 | .011 |
| | .0202 | | .01 | | .01 | .03 | .04 |
| Maximum Values | .0244 | .011 | [illegible] | [illegible] | [illegible] | [illegible] | .127 |

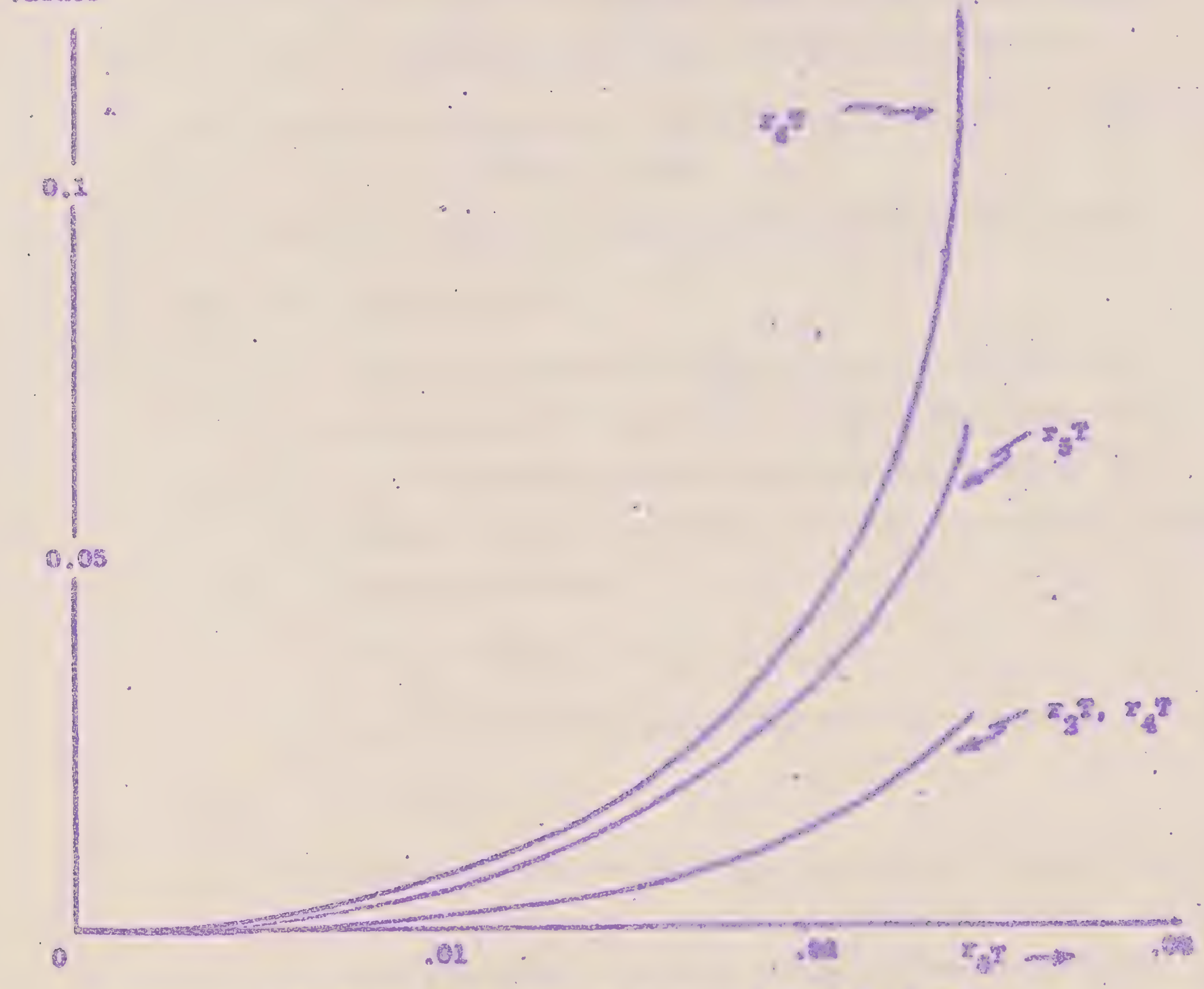


Fig. A-1. Relationship between $r_0T$, $r_2T$, $r_4T$
$r_5T$, $r_6T$ (Split Channel. No Cooperation)

## D. Explanations of the Results

The values of $r_0T$, $r_1T$, ...., in Table 2, and shown in Fig. 1, express channel utilization as a fraction of the available channel. Assuming that a finite channel of "unit" quantity is available for both the traffic going to and coming from the station, it was found* that in order that equal amounts of traffic may flow in both directions, about 2/3 channel must be allotted for traffic going to the station and 1/3 for traffic coming from the station. The reason, heuristically, is that traffic from the station is more organized and experiences less interference and therefore has less need of channel.

Considering now the set of maximum values and taking into acoout the 2/3 fraction of channel, we have the following observations:

(i) The total maximum useful traffic "$\Sigma r_0T$" from all 18 repeaters is: 2/3 (18 X .0244) = .293.

This is the number indicated in the summary of the results.

(ii) The a repeaters spend:

- A time proportional to $\{(r_0+r_5) + (r_0+ 1/3r_6)\}T$ in handling traffic from their own terminals. Note that $r_6$ is the number of total retransmissions from an a repeater, and 1/3 is assumed to be associated with traffic from own terminals.

  Let $T_1 = \{(r_0+r_5) + (r_0 + 1/3r_6)\}T$

  Substitution of maximum values from Table 2 gives

  $T_1 = .157$

- A time proportional to $\{(r_0+r_2) + (r_0+r_4) + 2r_0+ 2/3r_6\}T$ in handling transit traffic from two b repeaters. Let this time be $T_2$. Substitution of maximum values from Table 2 gives

---

*Refer to last part of this section for explanation.

$T_2 = .237$

3 A time proportional to $\{(r_0+r_4) + 2(3r_0+r_5)\}T$ in listening to one b repeater and two a repeaters whose traffic is not their responsibility. This is external interference. Again substituting maximum values,

$T_3 = .458$

Thus total active time at an a repeater is proportional to $(T_1+T_2+T_3) = .852$.

$T_1$ is about 20% of the active time

$T_2$ is about 28% of the active time

$T_3$ is about 52% of the active time

These are the numbers indicated in the summary of the results.

NOTE: The solution of the three equations relating to the traffic from the station was obtained in a manner similar to that used for traffic to the station. It was observed that the total transmissions from the station, i.e., $9(3r_0+r_7)T$, can at best be 1 (assuming unit channel availability). Using this fact, the maximum value of $r_0T$ for the traffic from the station was found to be .038. Comparing this with the maximum value of $r_0T$ for traffic to the station, i.e., .0244, it is easily seen that with a total available channel of unity the channel should be divided such that

$$\frac{.0244}{.0244 + .038} = .37 \text{ is allotted for traffic from the station}$$

and $$\frac{.038}{.0244 + .038} = .63 \text{ is allotted for traffic to the station}$$

The above fractions have been rounded off to 1/3 and 2/3 for simplicity.

Note also that under maximum operating conditions, the station spends a time proportional to "$2 n r_0 T$" = .538 per unit time in handling useful traffic ($n r_0 T$ for receiving and $n r_0 T$ for transmitting). Thus time spent by station in handling useful traffic is about 53% as indicated in the summary.

## Appendix B

## Equations and Their Solutions for Case 2

In this Appendix we consider Case 2. As for Case 1, we first develop the equations in Part A, summarize them in Part B, solve them in Part C and explain in Part D.

Case 2. Split Channel. Cooperative Repeaters

### A. Development of Equations

As before, let us first consider the traffic to the station and refer to Fig. 1.

- Consider $(r_0+r_1)$ attempting to reach $b_1$

  $b_1$ can listen to $(r_0+r_1)$ and will therefore not generate traffic simultaneous to $(r_0+r_1)$. However, $b_2$, $b_{12}$, and $a_1$ cannot listen to $(r_0+r_1)$. Then the conflicting traffic for $(r_0+r_1)$ is proportional to:

  $$R_1 = (r_0+r_1) + 2(r_0+r_4) + (3r_6+r_6) \tag{1}$$

  $$\text{and } r_1 = (r_0+r_1)\,[1 - e^{-R_1T}]$$

  $$\therefore\ r_0 = (r_0+r_1)\,e^{-R_1T} \tag{2}$$

- Consider $(r_0+r_3)$ attempting to reach $b_2$

  $b_2$ will cooperate. Conflicting traffic is from $b_1$, $b_3$, $a_1$, and $a_2$

  $$\therefore\ R_3 = (r_0+r_3) + 2(r_0+r_2) + 2(3r_6+r_6) \tag{3}$$

  $$\text{and } r_0 = (r_0+r_3)\,e^{-R_3T} \tag{4}$$

- Consider $(r_0+r_2)$ attempting to reach $a_1$

  Conflicting traffic is from $a_2$, $a_6$ and the terminals associated with $a_1$. Thus

$$R_2 = (r_0+r_5) + 2(3r_0+r_6) \tag{5}$$

$$\text{and } r_0 = (r_0+r_2)\, e^{-R_2 T} \tag{6}$$

- Consider $(r_0+r_4)$ attempting to reach $a_1$

  Conflicting traffic is from $b_{12}$, $a_5$ and the terminals associated with $a_1$. Thus

$$R_4 = (r_0+r_4) + (3r_0+r_6) + (r_0+r_5) \tag{7}$$

$$\text{and } r_0 = (r_0+r_4)\, e^{-R_4 T} \tag{8}$$

- Consider $(r_0+r_5)$ attempting to reach $a_1$

  Conflicting traffic is from $b_3$, $b_9$, $b_{12}$, $a_2$, and $a_6$.

$$\therefore\ R_5 = (r_0+r_5) + (r_0+r_2) + 2(r_0+r_4) + 3(3r_0+r_6) \tag{9}$$

$$\text{and } r_0 = (r_0+r_5)\, e^{-R_5 T} \tag{10}$$

- Consider $(3r_0+r_6)$ attempting to reach the station

  Conflicting traffic is from $a_3$, $a_4$, and $a_6$

$$\therefore\ R_6 = 3(3r_0+r_6) \tag{11}$$

$$\text{and } r_0 = \frac{(3r_0+r_6)}{3}\, e^{-R_6 T} \tag{12}$$

Equations 2, 4, 6, 8, 10 and 12 provide the relationship between $r_0$, $r_1$, .... $r_6$, for traffic going to the station with cooperative repeaters.

Considering now the traffic from the station and referring to Fig. 2.

- Consider $(3r_0+r_7)$ trying to reach $a_1$

  Conflicting traffic is from $b_1$, $b_2$ and $b_{12}$ only

  $$\therefore\ R_7 = (r_0+r_{11}) + 2(r_0+r_{12})$$

  However, as was established in Appendix A, $r_{10} = r_{11} = r_{12} = 0$

  $$\therefore\ R_7 = 3r_0 \tag{13}$$

  $$\text{and } r_0 = \frac{(3r_0+r_7)}{3} e^{-R_7 T} \tag{14}$$

- Consider the transmission from $a_1$

  Since the neighboring repeaters are cooperating, $a_1$ does not face any conflicts in its attempts to deliver its traffic to repeaters $b_1$, $b_2$, and the terminals. Thus, in this case $r_8$ and $r_9$ are also zero. Equation 14 thus is the only equation needed to describe the traffic away from the station.

B. <u>Summary of the Equations</u>

- For traffic going to the station

  $$r_0 = (r_0+r_1)\, e^{-[(r_0+r_1) + 2(r_0+r_4) + (3r_0+r_6)]T}$$

  $$r_0 = (r_0+r_3)\, e^{-[(r_0+r_3) + 2(r_4+r_5) + 2(3r_0+r_6)]T}$$

  $$r_0 = (r_0+r_2)\, e^{-[(r_0+r_5) + 2(3r_0+r_6)]T}$$

$$r_0 = (r_0+r_4)\, e^{-[(r_0+r_4) + (r_0+r_5) + (3r_0+r_6)]T}$$

$$r_0 = (r_0+r_5)\, e^{-[(r_0+r_5) + (r_0+r_2) + 2(r_0+r_4) + 2(3r_0+r_6)]T}$$

$$r_0 = \frac{(3r_0+r_6)}{3}\, e^{-3(3r_0+r_6)T}$$

e For traffic coming from the station

$$r_0 = \frac{(3r_0+r_7)}{8}\, e^{-3r_0T}$$

C. <u>Solution of the Equations</u>

Considering the equations defining the traffic to the stations,

let

a. $x_0 = r_0T$, $x_1 = (r_0+r_1)T$, $x_2 = (r_0+r_2)T$, $x_3 = (r_0+r_3)T$, $x_4 = (r_0+r_4)T$,

$x_5 = (r_0+r_5)T$, $x_6 = (3r_0+r_6)T$

The equations can then be written as

$$x_0 = x_1\, e^{-(x_1+2x_4+x_6)} \qquad \text{(C-1)}$$

$$x_0 = x_3\, e^{-(x_3+2x_2+2x_6)} \qquad \text{(C-2)}$$

$$x_0 = x_2\, e^{-(x_5+2x_6)} \qquad \text{(C-3)}$$

$$x_0 = x_4\, e^{-(x_4+x_5+x_6)} \qquad \text{(C-4)}$$

$$x_0 = x_5\, e^{-(x_5+x_2+2x_4+2x_6)} \qquad \text{(C-5)}$$

$$x_0 = \frac{x_6}{3}\, e^{-3x_6} \qquad \text{(C-6)}$$

Again equation (C-6) gives a simple relationship between $x_0$ and $x_6$ and a few set of values are:

| | | | | |
|---|---|---|---|---|
| $x_0$ | 0 | .0247 | .0408 | .0402 |
| $x_6$ | 0 | .1 | .33 | .4 |

Note that $x_0$ is maximum at $x_6 = \frac{1}{3}$.

Considering the equations (C-3), (C-4), and (C-5), it can be shown that if $x_5$ is eliminated from these equations, we can obtain the following two equations:

$$x_2 = x_4\, e^{(x_6 - x_4)} \qquad \text{(C-7)}$$

$$e^{2x_4} = \frac{e^{-x_2}}{x_2}\left(\log \frac{x_2}{x_0} - 2x_6\right) \qquad \text{(C-8)}$$

A few sets of values of $x_0$ and $x_6$ were substituted in (C-7) and (C-8) and the equations were solved for $x_2$ and $x_4$ by drawing the curves for (C-7) and (C-8) using $x_2$ and $x_4$ as variables and find the intersection points. Values of $x_2$ and $x_4$ were found for a few sets of values of $x_0$ and $x_6$. The calculation of $x_3$, $x_1$ and $x_5$ then is a straightforward process. In Table 1, values of $x_2$ and $x_4$ corresponding to a few values of $x_0$ and $x_6$ are indicated. The values of $x_1$, $x_3$ and $x_5$ for maximum value of $x_0$ are also indicated.

Table 1. $x_2$ and $x_4$ for a Set of Values of $x_0$ and $x_6$

| | $x_0$ | $x_6$ | $x_2$ | $x_4$ | $x_1$ | $x_3$ | $x_5$ |
|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | .0247 | .1 | .035 | .025 | — | — | — |
| Maximum Values | .0408 | .33 | .087 | .063 | .065 | .095 | .1 |

The corresponding values of $r_0$, $r_1$, $r_2$, $r_3$, --- $r_6$ are indicated in Table 2. The behavior of $r_3$, $r_4$, and $r_6$ as a function of $r_0$ is shown graphically in Fig. B-1.

Table 2. Values of Various $(rT)^*$ as a Function of $(r_0T)$

| $r_0T$ | $r_6T$ | $r_2T$ | $r_4T$ | $r_1T$ | $r_3T$ | $r_5T$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| .0247 | .03 | .011 | .006 | — | — | — |
| .0408 | .21 | .046 | .023 | .024 | .054 | .096 |

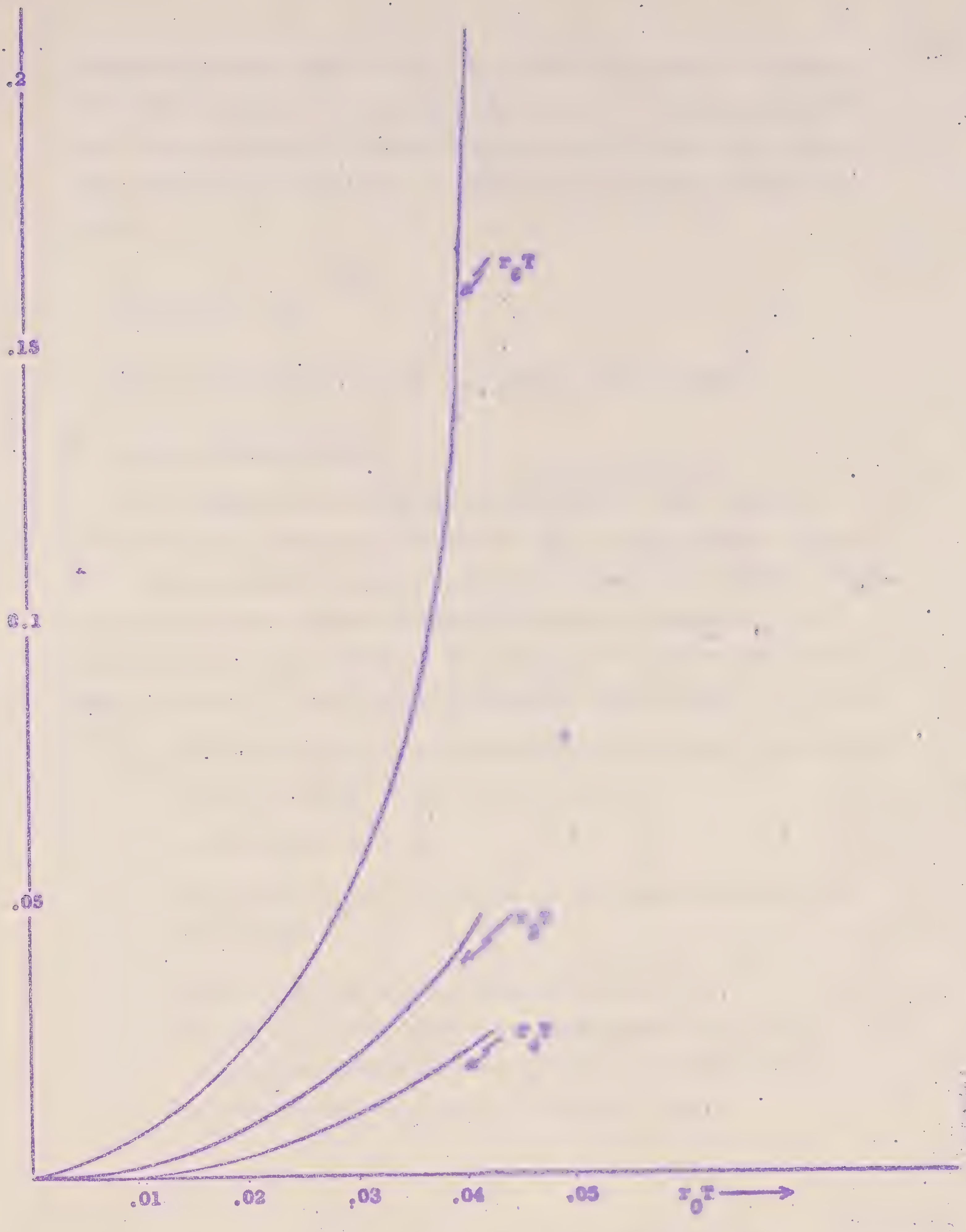


Fig. B-1. $r_2T$, $r_4T$ and $r_6T$ as a function of $r_0T$
(Split Channel. Cooperative Repeaters)

Considering now the equation for traffic from the station, we observe first that the station is transmitting a total of $6(3r_0+r_7)$ messages. For a unit channel availability $6(3r_0+r_7)T$ can be at the most unity. Thus, substituting $6(3r_0+r_7)T = 1$ (in the last equation of Part B), we get

$$\text{Maximum } r_0T = \frac{1}{18} e^{-3r_0T}$$

giving Maximum $r_0T \cong .044$ for traffic from the station.

D. <u>Explanation of Results</u>

The maximum values of $r_0T$ for traffic going in and coming from the station are respectively 0.0408 and .044 of the available channel, i.e., are approximately equal fractions of the available channel. Thus the available total channel should be divided in approximately two equal parts, one for traffic to the station and the other for traffic from the station. This is what is indicated in the summary of results.

(i) For a 1/2 channel availability in each direction, the maximum value of traffic "n $r_0T$" in the net is:

1/2 X 18 X .0408 $\cong$ .36

For n = 36, $r_0T$ = .01. These are the numbers indicated in the summary.

(ii) Each a repeater spends a time proportional to $[(r_0+r_5) + \frac{1}{3}(3r_0+r_6)]T$, i.e., proportional to $x_5 + \frac{x_6}{3}$ in handling traffic from own terminals. Substitution of maximum values for $x_5$ and $x_6$ from Table 1 give

$T_1 = .21$

Each a repeater spends a time proportional to $[(r_3+r_4) + 2(3r_0+r_5)]T$, i.e., $(x_4 + 2x_6)$, in listening to one b and two a repeaters whose traffic is not its responsibility, e.g., for $a_1$, the traffic from $b_{12}$, $a_2$ and $a_6$ is not its responsibility. Substituting the values,

$T_3 = .723$

$T_1$ is about 18%, $T_2$ about 28% and $T_3$ is about 54% of $(T_1 + T_2 + T_3)$. These are the numbers indicated in the summary.

The time spent by station in handling useful traffic:

$2n\ r_0 T = .72$, i.e., 72%

## Appendix C

## Equations and Their Solutions for Case 3
## (Split Channel. Total Cooperation Among "a" Repeaters)

### A. Development of Equations

We assume that while transmitting the traffic to the station, the six a repeaters cooperate perfectly, i.e., only one "a" repeater transmit at any one time. However, b repeaters operate independently. It is easily seen that "a" repeaters will not need to retransmit any messages since there is no conflicting traffic. Thus $r_s$ is zero. Furthermore, full available channel can be utilized by the 6 "a" repeaters. Thus

$$18\, r_0 T = \text{available channel.}$$

Now since no cooperation is assumed between "a" and "b" repeaters the first five equations given in Part B of Appendix A (Case 1) are applicable in the present case as well as with $r_s = 0$. The sixth equation is replaced by $18 r_0 T = 1$ (assuming unit channel). Also applicable are the 3 equations relating to the traffic from the station since the traffic from the station behaves in the same manner in the present case.

### B. Summary of the Equations

Refer to Part B of Appendix A and put $r_s = 0$ in the first five equations to obtain the equations for the present case. The sixth equation is replaced by

$$18 r_0 T = 1 \quad \text{(assuming channel availability to be unity).}$$

For the traffic from the station, the three equations in Part B of Appendix A are applicable here as well.

C. <u>Solution of Equations</u>

Substituting $r_0T = x_0$, $(r_0+r_1)T = x_1$, ———, etc., we obtain the following equations.

- For traffic going to the stations

$$x_0 = x_1 e^{-(x_1+x_2+2x_4+3x_0)} \qquad \text{(C-1)}$$

$$x_0 = x_3 e^{-(x_3+x_4+2x_2+6x_0)} \qquad \text{(C-2)}$$

$$x_0 = x_2 e^{-(x_5+2x_4+9x_0)} \qquad \text{(C-3)}$$

$$x_0 = x_4 e^{-(x_4+x_5+x_2+9x_0)} \qquad \text{(C-4)}$$

$$x_0 = x_5 e^{-(x_5+2x_4+x_2\,9x_0)} \qquad \text{(C-5)}$$

$x_0 = \frac{1}{18}$ (maximum value for traffic going to the station)

Equations (C-3), (C-4) and (C-5) can be combined to obtain:

$$x_2 = x_4$$

and $x_2 e = \log x_2 - \log x_0 - 2x_2 - 9x_0$.

For $x_0 = \frac{1}{18}$, values of $x_2$, $x_4$, $x_5$ and then for $x_1$ and $x_3$ were calculated in a manner similar to that indicated in Appendix A. These are:

$$x_0 = \frac{1}{18} = .0555$$

$$x_1 = .12,\ x_3 = .14,\ x_2 = x_4 = .16,\ x_5 = .26$$

In terms of $(rT)^s$, the values are:

$r_0T = .0555$, $r_1T = .065$, $r_3T = .085$

$r_2T = .105$, $r_4T = .105$, $r_5T = .194$

Now considering the traffic from the station, the maximum value of $r_0T$ was found to be .038 as explained in Part B of Appendix A. We shall use the same value for the present case also.

D. <u>Explanation of Results</u>

The maximum value of $r_0T$ for the traffic going to the station is .055 of the available channel and that for traffic going from the station is .038 as indicated above. Therefore, to equalize the useful traffic in both directions, the total available channel should be divided so that:

$$\frac{.038}{.038 + .055} = .4 \text{ is allotted to the traffic to the station}$$

and

$$\frac{.055}{.038 + .055} = .6 \text{ is allotted to the traffic from the station}$$

0.4 and 0.6 have been expressed as 2/5 and 3/5 in the summary of the results

(i) The total useful traffic "$n\,r_0T$" for the net is given by:

$$2/5 \times 18 \times .0555 = .4;$$

For $n = 36$, $r_0T = .011$

(ii) Each a repeater spends a time proportional to $T_1 = [(r_0+r_5) + (r_0)]T$ to handle traffic from own terminals. Using maximum values,

$$T_1 = .3$$

Each a repeater spends a time proportional to $T_2 = [(r_0+r_4) + (r_0+r_2) + 2r_0]T$ to handle traffic (transit) from $b_1$ and $b_{12}$. Substituting maximum values

$$T_2 = .43$$

Each a repeater spends a time proportional to $T_3 = [(r_0 + r_4) + 2(3r_0)]T$ in listening to two a repeaters and one b repeater whose traffic is not its responsibility. Substituting the values;

$$T_3 = .5$$

$T_1$ is about 24%, $T_2$ is about 35%, and $T_3$ is about 41% of $T_1 + T_2 + T_3$ as indicated in the summary of results.

The time spent by station in handling useful traffic:

$$2\ n\ r_0 T = .8,\ \text{i.e.},\ 80\%$$

## Appendix D

### Equations and Their Solutions for Case 4
### (Single Channel for Traffic to and from the Station)

### A. Development of Equations

The equations for a two ring network for this case tend to become quite involved and do not lend themselves to an easy solution. Also, it was established during the studies of the first three cases that the maximum value of "n $r_0T$" (the total useful messages in the net) can be calculated considering only the single ring of repeaters. Thus it was decided to consider only a 6 repeater network for this case to at least find out the maximum value of n $r_0T$ without going through complicated calculations.

Referring to Fig. D-1, let $a_1$, $a_2$, ——— $a_6$ be the six repeaters arranged in a hexagonal pattern around the station. Let

$r_0$ = Total useful traffic offered and received by terminals in each repeater area

$r_1$ = Total retransmissions from terminals of each repeater area

$r_2$ = Total retransmissions from each a repeater in the direction to the station

$r_3$ = Retransmissions from the station per repeater, i.e., total retransmissions = $6r_3$

$r_4$ = Total retransmissions from each repeater to its terminal.

This traffic pattern is indicated in Figure D-1. We assume that all "a" repeaters behave identically.

- Consider $(r_0+r_1)$ attempting to reach $s_1$

The conflicting traffic is:

$$R_1 = (r_0+r_1) + 5(r_0+r_2) + 3(r_0+r_4) + 6(r_0+r_3) \tag{1}$$

$$\therefore \quad r_1 = (r_0+r_1)[1 - e^{-R_1 T}]$$

$$\therefore \quad r_0 = (r_0+r_1)e^{-R_1 T} \tag{2}$$

- Consider $(r_0+r_2)$ attempting to reach S

The conflicting traffic is:

$$R_2 = 5(r_0+r_2) + 6(r_0+r_3) + 3(r_0+r_4) \tag{3}$$

$$\therefore \quad r_0 = (r_0+r_2)e^{-R_2 T} \tag{4}$$

- Consider $(r_0+r_3)$ attempting to reach one "a" repeater

The conflicting traffic is:

$$R_3 = (r_0+r_1) + 3(r_0+r_4) + 3(r_0+r_2) \tag{5}$$

$$\therefore \quad r_0 = (r_0+r_3)e^{-R_3T} \tag{6}$$

- Consider $(r_0+r_4)$

The conflicting traffic is:

$$R_4 = (r_0+r_1) \tag{7}$$

$$\therefore \quad r_0 = (r_0+r_4)e^{-R_4T} \tag{8}$$

B. Summary of Equations

- $r_0 = (r_0+r_1)e^{-[(r_0+r_1) + 3(r_0+r_2) + 3(r_0+r_4) + 6(r_0+r_3)]T}$

- $r_0 = (r_0+r_2)e^{-[5(r_0+r_2) + 6(r_0+r_3) + 5(r_0+r_4)]T}$

- $r_0 = (r_0+r_3)e^{-[(r_0+r_1) + 3(r_0+r_4) + 3(r_0+r_2)]T}$

- $r_0 = (r_0+r_4)e^{-(r_0+r_1)T}$

C. Solution of Equations

Define: $x_0 = r_0T$, $x_1 = (r_0+r_1)T$, $x_2 = (r_0+r_2)T$, $x_3 = (r_0+r_3)T$,

$x_4 = (r_0+r_4)T$

The equations can then be written as:

$$x_0 = x_1e^{-[x_1+3x_2+3x_4+6x_3]} \tag{C-1}$$

$$x_0 = x_2 e^{-[5x_2+6x_3+5x_4]} \quad \text{(C-2)}$$

$$x_0 = x_3 e^{-[3x_2+3x_4+x_1]} \quad \text{(C-3)}$$

$$x_0 = x_4 e^{-x_1} \quad \text{(C-4)}$$

The above equations were solved by assuming various values of $x_0$ and solving for $x_1$, $x_2$, $x_3$ and $x_4$ using Newton's Method. The maximum value of $x_0$, for which solutions for $x_1$, $x_2$, --- , could be obtained was .032. The corresponding values for $x_1$, $x_2$, --- , are:

$x_0$ = .032

$x_1$ = .062

$x_2$ = .072

$x_3$ = .047

$x_4$ = .034

The corresponding values in terms of "rT"s are:

$r_0T$ = .032

$r_1T$ = .03

$r_2T$ = .04

$r_3T$ = .015

$r_4T$ = .002

D. Explanation of Results

(1) The maximum value of $r_0T$ per repeater area as calculated above is based on a 6 repeater net. Thus the total useful traffic for the net "n $r_0T$" = 6 X .032 = .192. This is the value indicated in the summary of results. If there were 36 repeaters

in the net, then 36 rT would have been equal to .192 and $r_0T$ per repeater would be .0053, as indicated in the summary.

(ii) The total traffic at each "a" repeater associated with its own terminal is proportional to:

$$(r_0+r_1)T + (r_0+r_2)T + (r_0+r_3)T + (r_0+r_4)T = .215$$

The external traffic at each "a" repeater is proportional to:

$$5(r_0+r_3)T + 2(r_0+r_4)T = .303$$

Note that the own traffic at an a repeater (proportional to .215) would have been the sum of own plus transit if we had considered a two or three ring of repeaters. Thus the fraction of active time spent by an a repeater in handling own (plus transit) traffic is $= \frac{.215}{.215 + .303} \cong 41\%$

The rest of the active time, i.e., 59% is spent in listening to external traffic. This is the result indicated in the summary.

The time spent by the station in handling useful traffic:

$2n\ r_0T = .384$, i.e., about 38%